correction for continuum background was made; moreover, the upper levels of both lines have approximately the same energy (35,257 cm^{-1} and 36,686 cm^{-1}) so that the accuracy of determining the temperature is low with $\Delta T/T \sim (E_1 - E_2)^{-1}$. Indeed, the number density of neutral atoms derived from the two lines differs by about a factor 2, the stronger line giving the smaller number density.

The number density of the iron ion has been computed from the Saha equation assuming singly ionized atoms;

$$N^{+2}/N = (2U^+/U)(2\pi mkT/h^2)^{3/2}e^{-E^+/kT} \quad (2)$$

where the superscript denotes the quantities for the ionized atom.

The measurement for Cr density was performed in analogous way. The line observed at 4289.7 Å results from transition to the ground state, and absorption by the (cold) free-stream may be expected. This absorption, however, is estimated[6] to be less than 30% for the total line intensity.

For the CN density measurement, the intensity of the band at 3779 Å has been measured with an effective exit slit of 7.5 Å.

For the emission from a molecular band, again for optically thin gas, we may write for the intensity [erg sec^{-1} cm^{-2} sr^{-1}]

$$I = K_\lambda l 2hc^2\lambda^{-5}e^{-hc/kT\lambda}\Delta\lambda \quad (3)$$

the absorption coefficient K_λ is given by

$$K_\lambda = (\pi e^2/mc)f\varphi N(h/kT)e^{-hc/kT(1/\lambda_{00} - 1/\lambda)} \quad (4)$$

φ is a dimensionless factor of order unity, which gives the details of the rotational spectrum. We set $\varphi \equiv 1$ for simplicity and in view of the uncertainty of the electronic f number for the CN violet band.

The oscillator strength for the CN violet band is not well known. Soshnikov,[7] in a review article, cites White who found $f = 0.1 - 0.06$; he tentatively proposes $f = 0.086$; Herzberg[8] gives $f = 0.026$. We choose the value $f = 0.086$ which is in agreement with a theoretical estimate of Mulliken, who obtained $f \sim 0.1$.§

Equation (3) with Eq. (4) may be solved for the number density N. The atomic lines as well as the band segment for CN have been selected to give minimum mutual interference; the measured intensities have not been corrected for background radiation, nor has a wing correction been applied for the line intensities. With effective exit slit of 2 Å and a half width of ~0.2 Å, this correction is small.

Figure 1 gives two examples of the radiation from the impurities. All the traces were either of the type where high Fe radiation occurs before or during testing time or of the type where Fe radiation was relatively low during testing time with high values at later times. The maximum CN radiation always occurred before or during test time. The relatively high Cr radiation is probably a peculiarity of this particular tube, which has a chrome-plated section at the secondary diaphragm station. Table 1 summarizes the results of the impurity density measurements.

Inspection of the table shows that with an air number density of $>3 \times 10^{18}$ particles/cm^3 the total impurity is ~400 ppm for the most severe case, which is substantially lower than impurity levels from, say, seeding in MHD facilities. It is not significant in chemical nonequilibrium studies such as have been reported in Refs. 9 and 10, where the temperatures in the shock layer are quite high. However, these impurity levels can make a significant contribution to the radiative properties of the air impurity mixture.

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§ A recent unpublished survey suggests a value "of about 0.025" as being consistent with experimental and theoretical results. This value would increase the CN number density by a factor ~3.

Some Experiments on the "Spoke" Phenomenon in MPD Operation

F. MAISENHÄLDER,* E. UNGERER,† AND G. KRÜLLE*

Deutsche Versuchsanstalt für Luft- und Raumfahrt, Institut für Plasmadynamik, Stuttgart-Vaihingen, Germany

Introduction

THE problem of the appearance of a current spoke in MPD operation has attracted an increasing interest. It is the question of whether the current transfer to the anode occurs in an axisymmetric way or whether the current, or part of it, concentrates in a spoke that rotates, in other words, whether a more or less significant rotating disturbance exists.

Lovberg et al.[1] and Larson[2] have made particular experiments on this matter using electric and magnetic probes and a segmented anode, respectively. Connolly et al.,[3] among other data, report results on the spoke phenomenon, and so does Malliaris[4] in a recent paper. Qualitatively, the phenomenon has also been stated by Brockman et al.[5] Years ago, Hess[6] made similar observations in operation of a linear Hall accelerator.

All authors mentioned have found the existence of a total or partial spoke at nearly all operating conditions studied. Results concerning relative amplitude and rotation frequency of the spoke, however, are rather contradictory. Also, the question of whether the spoke is typical for MPD operation and, if it is, how its existence will affect thrust production has been left open.

The situation is such that more experimental evidence is needed. For this reason, the standard device operated in this institute, X9/40-16, was employed applying two methods: 1) a segmented anode, and 2) image converter records. Argon and helium were used as propellant gases.

Segmented Anode Measurements

For these experiments, thruster X9/40-16 having a nozzle diameter of 16 mm and an anode diameter of 40 mm as

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* Senior Research Scientist.

† Research Scientist.

described in Ref. 7 was supplied with an anode ring separated into two azimuthal halves. The alternating portion of the current flowing through either segment was measured by use of Rogowski coils surrounding the two positive current leads.

Provided that $t \ll RC$ where t is the oscillation period of the lowest frequency appearing, combination of a Rogowski coil with a passive R,C-integrating unit simply yields

$$U = R_S \cdot i$$

where U is the integrator output voltage, i the current measured, and $R_S = L/RC$ the impedance of the Rogowski coil. R_S was $1.2 \cdot 10^{-3}\Omega$ in our experiments.

If a rotating spoke does not exist which, according to former investigations,[8] seems to be the case when the external field is zero (compare also Ref. 4), the current through either segment equals $I/2$ where I is the total anode current. In the case of a spoke,

$$I = 2I_0 + I_S$$

where $I_0 < I/2$ is the constant portion of I going to either segment and I_s is the spoke current that alternatively charges the segments due to the rotation.

The peak value $\hat{\imath}$ of the alternating current $i = f(t)$ measured by Rogowski coils is related to the spoke current as

$$I_s = 2\hat{\imath}$$

In the limiting case, when the total current is in the spoke, $I = I_s = 2\hat{\imath}$.

Experiments were carried out under the following operating conditions: ambient pressure $p_\infty = 0.5$ mm Hg, argon flow rate $\dot{m}_A = 0.08 - 0.23$ g/sec, helium flow rate $\dot{m}_{He} = 0.022 - 0.06$ g/sec, current $I = 200 - 1000$ amp, and magnetic field strength (at cathode tip) $B = 0 - 2500$ gauss.

Oscillograms of the two Rogowski coil outputs show that alternating currents are going to the anode halves. A phase difference of 180° occurs, implying existence of a rotating spoke. Plots taken without an integrating unit show a superimposed fast oscillation having a frequency of about 1.3 Mcycles independent of variable operation parameters. The current portion calculated from the amplitude of those high-frequency disturbances, however, is of the order of only 1% of the total current so they will not be considered further. For reasons of a precise measurement of the low-frequency main portion of current, they have been suppressed, in the later experiments, by use of an integrator.

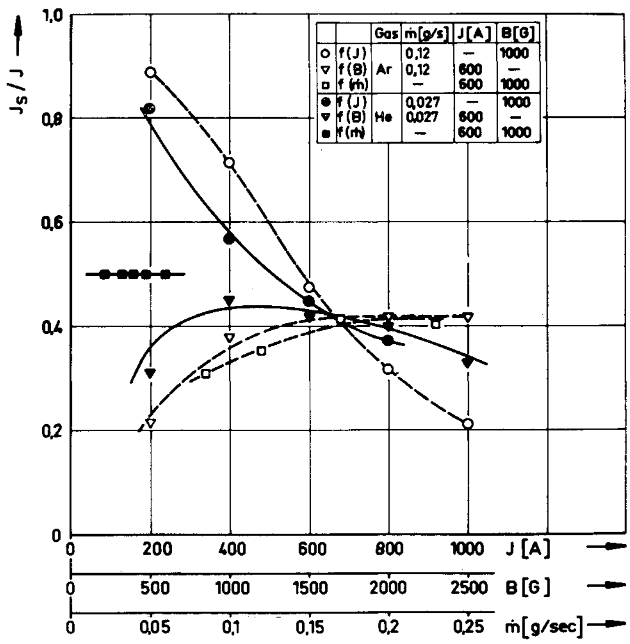

Fig. 1 Relative spoke current as a function of main parameters at $p_\infty = 0.5$ mm Hg.

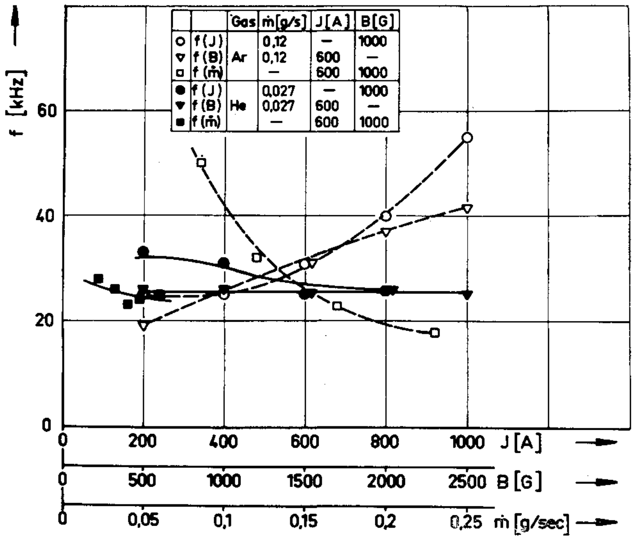

Fig. 2 Rotation frequency as a function of main parameters at $p_\infty = 0.5$ mm Hg.

In Fig. 1, the relative spoke current I_s/I is shown as a function of the main parameters. It is seen that, at any rate, only part of the current is carried by the spoke. The spoke portion is a strong function of current, however, and approaches unity (total spoke) at the lowest current applied (in this case $B = 1000$ gauss). I_s/I is not a clear function of B and $\dot{m}$. Both gases show similar tendencies, though variations are more pronounced in argon.

Figure 2 shows the rotation frequencies as a function of the parameters. In argon, the frequency increases with increasing current, with increasing magnetic field (in both cases somewhat weaker than linear), and with decreasing mass flow. In helium, the influence of variations is poor. Thus, the results obtained with the two gases are not to be correlated. Yet, all frequencies are in the range of 20 to 50 kcycles.

Image Converter Records

For a qualitative study of the spoke phenomenon an image converter was used. It was adjusted to an exposure time of $2 \cdot 10^{-7}$ sec. As expected, with no external field applied, no deviation from axisymmetry of the thruster exhaust was optically observed. However, also in cases of considerable external field strengths, the plume appearance was not visibly disturbed. Figure 3 shows a helium plume at such an operating condition when a relative spoke current I_s/I of about 30% was measured (the slanted structure in the record is due to a disturbance of electro-optical projection by the converter tube gate.)

The only case when a spiral form, i.e., serious nonaxisymmetry of the exhaust was recorded was when cooling water

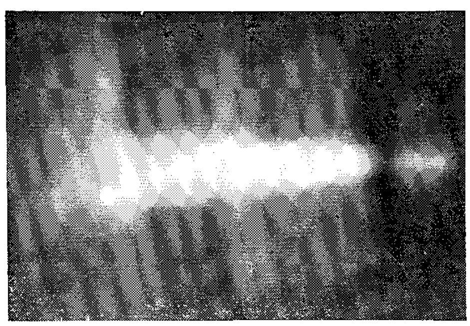

Fig. 3 Image converter record, exposure time 2.10^{-7} sec. Helium plume at $\dot{m} = 0.033$ g/sec, $I = 1000$ amp, $B = 1350$ gauss.

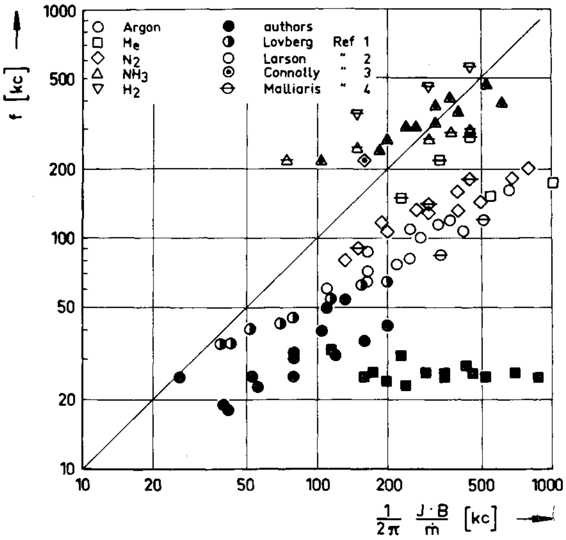

Fig. 4 Comparison of rotation frequencies as a function of mass rotation parameter $(I \cdot B)/2\pi\dot{m}$.

leakage of the anode occurred. It may be stated that no definite spoke structure was found using the image converter. A spoke portion of current, at least if moderate, does not clearly show in the transient optical appearance of the exhaust plume.

Discussion

The experiments show that there exists, within the discharge, a region of increased current density which may be interpreted as a rotating spoke. It may be concluded from the oscillograms, however, that this spoke has a large azimuthal spread and, therefore, its structure is diffused. This result is consistent with the converter records. The spoke portion of current has a tendency to vanish at high currents which corresponds to observations made by Ref. 4 but is in contrast to results of Refs. 1 and 2, which report a total spoke in all cases. These discrepancies may arise from the different ranges of ambient pressure in which the experiments were performed.

For further comparison of experimental evidence, data of rotation frequencies as plotted previously were compiled in Fig. 4 together with results of Refs. 1–4, as a function of $(I \cdot B)/2\pi\dot{m}$. Choice of this term as an abscissa is made assuming that a correlation of spoke rotation with mass rotation of the exhausted gas exists. Mass rotation frequency, under some simplifying assumption, may be calculated from magnetic torque yielding equality to the aforementioned parameter. Coincidence of both frequencies is best accomplished in case of a total (ion current) spoke "snowplowing" around according to a model described by Lovberg.[9] In case of a rotating disturbance, the connection is obviously more difficult; yet, at one operating point (argon 0.22 g/sec, 800 amp, 1200 gauss), coincidence was actually found comparing "electric" rotation frequency with ion mass rotation frequency as measured spectroscopically.[7]

In fact, tendency of spoke rotation frequency to rise with current and B field, as stated previously in case of argon, is reported by all authors reviewed. Logarithmic plotting yields $f \sim I^m \cdot B^n$, where both m and n are between 1.0 and 0.5. It is a striking fact that this dependance is similar to the behaviour of thrust as a function of the parameters (see Ref. 7 among others) which implies another connection between spoke and mass motion as thrust is produced, to a large extent, by the deflection of azimuthal to axial velocities.

Figure 4 shows that frequencies measured in this laboratory are lowest, but argon data correspond, in tendency described previously, to data presented by Refs. 1 and 2. Helium frequencies, however, are inconsistent with the general picture. Preliminary spectroscopic observations made in this laboratory[10] add to the explanation of helium anomaly. These are that a considerable amount of entrained ambient mass is found in the plume, that the degree of He ionization is probably very low, and that the rotational velocities of the ionized and neutral species differ strongly and are relatively small as compared to argon results; in fact, the neutral sort is almost at rest (azimuthally).

As an over-all result no satisfactory correlation of all accessible data could be achieved. Comparison of operating conditions shows that this is not simply due to differences of geometry and ambient pressure. We agree to the opinion[3,5] that there may be a strong influence on the rotation frequency of, among other parameters, degree of ionization and interaction between particle sorts, respectively.

It is to be concluded from our experiments that a spoke must not necessarily exist. Transition to the spoke from axisymmetry seems to happen gradually as destabilizing effects grow. Also we doubt whether the appearance of a rotating disturbance in current transition will necessarily entail serious deviation from axisymmetry of the downstream regions.

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Spectroscopic Temperature Measurements of Shock-Heated Helium

THOMAS GARLAND MCRAE, JR.*
University of Florida, Gainesville, Fla.

Nomenclature

E	= energy level
E_∞	= ionization energy
$\langle E \rangle$	= thermal average of excitation energy
e	= electronic charge

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* Research Associate, Department of Aerospace Engineering.